perspective

a new system for designers  by Jay Doblin
To Read Malb
Introduction

The serious designer, faced with the problem of solidifying and transmitting design ideas, finds no single tool more effective—or more economical—than skill in perspective drawing. But it is an awkward tool, for the traditional methods of drawing in perspective are complicated and time-consuming. What is more important, they are often result in inaccurate drawings.

These traditional systems were developed largely from the needs of the architect, who generally develops his ideas in plan, projecting drawings from the plan when his design is completed. A plan view of a building or a house is obviously not very informative, however. The industrial designer must work out his ideas in the round, and although he can work with models or mock-ups, this is slow and costly. For him, perspective is not simply a means of communication but a working tool as indispensable as the architect’s plans. He wants a simple method of creating on a two-dimensional surface an accurate illusion of a three-dimensional object—a system that does not depend on plan views or elaborate constructions.

Generally speaking, there are two ways of drawing in perspective. One depends on the trained eye to judge convergence, depth, etc., and is usually called free-hand drawing, although drafting instruments may be used. The other involves the use of one of the constructional systems, and is commonly referred to as mechanical perspective.

During the four years that I served as chairman of the Evening School of Industrial Design at Pratt Institute, I became increasingly dissatisfied with the two methods in general and the mechanical systems in particular. Both are important, and both were taught in the Evening School, but we began with the free-hand system because the over-all training was intended to sharpen the students’ vision in preparation for design work.

We found that the free-hand system served the gifted students well but that their temperaments, when exposed to the tedium of the mechanical systems, often prevented them from making accurate drawings. The less talented students, who were unable to draw perspectives by eye, made poor mechanical perspectives because the systems themselves were deficient. Moreover, the mechanical systems did nothing to encourage their free-hand skill. This is not, unfortunately, just a student problem; similar difficulties arise in professional offices.

Badly drawn perspectives are almost without exception the result of three fundamental errors, all of which are inherent in the traditional perspective systems:

1. The angle of vision may exceed the limits of accurate drawing.
2. Because the systems are complex and tedious, the margin of error in the final drawing is multiplied.
3. The designer often fails to predict accurately the view, size, or scale of the drawing.

The number of perspective views that can be drawn of a single object is, of course, infinite, varying with each position of the observer. Most perspective systems are based on unlimited cases of perspective, but the one I will present is based on just three technical relationships of the observer and the object. It has unlimited applications, however: by using these three situations with their few clear rules, it is possible to draw any perspective view with the basic drafting instruments. The system has these important advantages:

1. It results in photographic accuracy.
2. It allows easy predetermined of the view, scale, and size of the drawing.
3. It encourages free-hand skill.

Any perspective system looks complicated in print. I suggest that you try this new system at your drafting board to see how simple it is.
Basic principles of linear perspective

The basic principle: We are all away to diminish and from us nearer drawing is a web of reality that objects don’t readily understand maintain their perspective. Introducing my drawings.

There are various perspective, but a same underlying:
1. A perspective of reality by no and the picture
2. The observer who, with one eye, observes the object. The eye is at
3. The surface is called the plane of the object at the line of sight. The object and intersect the plane.
4. A perspective these points an do if the plane object are rays of light traced directly, plane is a piece board, and the project on it systems or visit.
5. The horizon infinite distance parallel lines in objects on the plane. These points a
6. Although in concerned with for convenience lines and plane and planes dim.
7. The cube is used and will be used most of the text. A perspective cube. The depth cube is the drawing inc.
The basic principles of linear perspective
We are all aware of the way objects appear to diminish and converge as their distance from us increases. Stated simply, perspective drawing is a way of reproducing this appearance of reality on a flat plane. Since we know that objects do not really diminish and can readily understand drawings in which they maintain their true proportions, practical perspective might be described as a way of introducing systematic distortions into drawings.

There are various methods of drawing in perspective, but all of them are based on the same underlying principles:
1. A perspective drawing creates the illusion of reality by relating the observer, the object, and the picture plane.
2. The observer stands in a fixed position and sees with one eye, like a camera. The position of the eye is called the station point.
3. The surface on which the drawing is made is called the picture plane, and is assumed to be a plane placed between the observer and the object at right angles to the observer’s line of sight. Lines drawn between the object and the observer’s eye will intersect the picture plane at various points.
A perspective drawing is made by plotting these points and connecting them. This is easy to do if the picture plane is a piece of glass; then the lines between the observer and the object are rays of light; the image on the glass is a perspective view and can be traced directly. Usually, however, the picture plane is a piece of paper on a drafting board, and the perspective drawing must be projected on it by one of the perspective systems or visualized and sketched free hand.
4. The horizon is assumed to be at an infinite distance from the observer, so that parallel lines meet at the horizon and objects on the horizon appear to be points. These points are called vanishing points.
5. Although in perspective drawing we are concerned with the diminishing of objects, for convenience we break these objects into lines and planes and study the way the lines and planes diminish.
6. The cube is the basic form in perspective, and will be used as the object throughout most of the text, because it can be used as a perspective unit to measure height, width, and depth concurrently. Theoretically, the perspective cube can be multiplied and divided into any combination of height, width, and depth to provide a basis for drawing any object.

Current texts set forth three basic working methods for constructing perspective drawings. Before proceeding to the new system, it will be helpful to review these three traditional systems. I shall not present a complete description of them, but simply show how they are used to erect a basic perspective cube. These systems are all accurate for drawing the cube; they admit inaccuracies as the scope of the drawing increases.
The two elevation system
The perspective drawing is derived from two orthographic drawings, a top view and a side view, which must be projected through two picture planes toward two observers.

Advantages: The system is useful if top and side views have already been made.

Disadvantages: The amount of drawing is time-consuming and conducive to error and confusion. If the station points are remote the construction will be unwieldy. The vanishing points cannot be located until the drawing is completed. Size, view, and scale are difficult to foresee. When orthographic drawings exist, the side view must usually be redrawn at the proper rotation. If drawings do not exist, the labor required is prohibitive.

The top plan system
The top plan system is easier to understand and faster because it requires only a top view and vanishing points.

Advantages: The system is useful if a plan view has already been made. It is less complex than the two elevation system and easier to visualize. It uses an actual-scale height measurement. It provides the vanishing points immediately.

Disadvantages: The location of a station point and long parallels can be cumbersome.

The measuring point system
The sides of the cube are derived from vanishing points; the depth of the cube is established by the use of measuring points, which are actually the vanishing points of lines relative to the cube.

Advantages: No plans or elevations are necessary. The cube is easily multiplied directly from the basic construction.

Disadvantages: The theory is complex (below). Construction of the station points and large right angle is cumbersome.

To locate the measuring points of a cube
1. Draw a top view above of the cube and place it against the horizon at d.
2. Drop a vertical at d and locate SP.
3. Draw parallels to the sides of the cube from SP to locate VP-L and VP-R.
4. Measuring points MP-A and MP-B can be located as follows: Rotate point a from d to the horizon, locating point A; draw a line from A through s; draw a parallel to As from SP to the horizon, locating MP-A; this is the vanishing point of line As. Repeat for point b, locating MP-B. Since Asa and VP-L are equal angles because their sides are parallel, we can find MP-A and MP-B simply by rotating SP from VP-L and VP-R.
5. Find the nearest angle n of the cube by measuring any desired distance from d.
6. Draw perspective lines from s to construct the nearest angle.
7. Draw a measuring line horizontally through s.
8. Mark the length of the cube side on either side of s, locating s, and p.
9. Draw the line Ap in perspective by connecting s with MP-A. Where this line intersects the line from a corner of the cube will appear. Repeat for Bs. We now have all the points necessary to complete the cube.

To draw a cube
1. Draw a top view against the picture plane.
2. Locate a station point perpendicular at distance in relation to SP.
3. Connect SP-T note their intersections.
4. Draw a side view against a side view.
5. Locate eye level distance along the top (a) of S.
6. Locate the side of SP-S note their intersections.
7. Drop all points horizontally.
8. Extend all points horizontally.
9. Cross-reference between the top and cross-referencing perspective and continue.
To draw a cube using the two elevation system:
1. Draw a top view $T$ of the cube and place it against the picture plane at $x$.
2. Locate a station point $SP-T$ by dropping a perpendicular at $x$ and scaling off any desired distance in relation to the cube size.
3. Connect $SP-T$ to all important points on $T$ and note their intersections with the picture plane.
4. Draw a side view $S$ of the cube and place it against a side view of the picture plane.
5. Locate eye level $y$ by measuring any desired distance along the side view picture plane from the top ($y$) of $S$.
6. Locate the side view station point $SP-S$ by drawing a line horizontally from $y$ equal to the distance between $SP-T$ and the top view picture plane.
7. Connect $SP-S$ to all important points on $S$ and note their intersections with the picture plane.
8. Drop all points on the top view picture plane.
9. Extend all points on the side view picture plane horizontally.
10. Cross-reference all points on the grid formed between the top and side views of the cube (note cross-referencing of points A to locate A in perspective) and connect them properly to form cube.

To draw a cube using the top plan system:
1. Draw a horizontal line to serve as horizon and picture plane.
2. Draw a top view of the cube at any angle with nearest corner touching the picture plane at $x$.
3. Locate the station point $SP$ by dropping a perpendicular from the horizon at $x$ and scaling off any desired distance in relation to cube size.
4. From $SP$ draw lines $a$ and $b$ parallel to sides $A$ and $B$ to intersect the horizon at $vp-l$ and $vp-r$.
5. These points are the vanishing points.
6. Connect $SP$ with all important points of the cube and note the intersections at the picture plane (3, 8, and 4).
7. Drop verticals from 8, 9, and 9.
8. The distance of the perspective cube from eye level is laid off on $SP-x$, locating the nearest angle $N$ of the cube. $xN$ cannot exceed $SP-x$.
9. Lay off the height of the cube from $N$ toward $x$.
10. Draw perspective lines from the bottom and top of the cube height to $vp-l$ and $vp-r$.
11. At the intersections of the perspective lines and the verticals draw the remaining perspective lines to complete the cube.

To draw a cube using the measuring point system:
1. Draw a horizon.
2. Set up two vanishing points $vp-L$ and $vp-R$.
3. Drop a vertical from any point on the horizon, depending on the view desired.
4. Construct a right triangle with the distance between $vp-L$ and $vp-R$ as its hypotenuse and its apex on the vertical locating station point $SP$.
5. Divide the vertical to any scale according to the distance of the observer from the object and place the nearest angle $N$ at any desired distance from eye level.
6. Draw measuring line $ml$ horizontally through $N$.
7. Lay off the true height of the cube from $N$ to $x$ and rotate it to the measuring line, locating $x$ and $y$.
8. Draw perspective lines to $N$, forming the nearest angle of the cube.
9. Locate measuring points $mp-Y$ and $mp-X$ by rotating $SP$ from $vp-L$ and $vp-R$.
10. Connect $mp-X$ to $x$ and note its intersection with the perspective line from $N$.
11. Repeat for $mp-Y$ and $y$.
12. Draw verticals at the intersections and complete cube by drawing the remaining perspective lines.
Introduction to 45° perspective

The traditional perspective system was constructed and extended in a way that would allow for viewing cubes at an angle. The simple oblique view can be constructed by adding a starting point.

45° oblique

To understand the case of the floor at 45° direction. 

1. The diagonal range of visibility on the horizon. This point is called the vanishing point or DP.
2. The side angles to the perspective view.
3. If the eye of sight (A) is at a side to side, parallel to the important lines below.

Proof that the perspective is correct:

Given the lines AB, BC intersect at D. 

1. Extend AB.
2. Since BE is parallel to AB, we know that it bisects each.
3. Since BE the angle R equals to C.
4. Similarly, they divide the segment.
5. Therefore, lines AB and BC are parallel.
The traditional systems we have just reviewed give general rules that can be used to develop any perspective view. However, in the opening section it was mentioned that an accurately constructed cube can theoretically be multiplied and divided indefinitely. This means that if we can find any views of the cube that are particularly easy to draw, we should be able to multiply these basic cubes to provide cubes at every angle to the observer. The simplest view of the cube is the 45° oblique view. Because this view is easy to construct and lends itself to the development of additional information, it will be used as a starting point for our discussion.

45° oblique perspective
To understand 45° oblique perspective, examine the case of an observer standing on a tile floor that extends to the horizon in every direction. He cannot see all of this vista at once. If he turns so that his line of sight is at an angle of 45° to the sides of the tiles, he will notice several things:
1. The diagonals of all the tiles within his range of vision seem to meet at a single point on the horizon directly in front of him.
   This point will be called the diagonal vanishing point or DVP.
2. The sides of the tiles converge at equal angles to the right and left toward their respective vanishing points (VP-L and VP-R).
3. If he examines one tile directly in his line of sight (AXBY) he sees that the side to side diagonal (AB) is horizontal and parallel to the horizon. This fact is extremely important to our system. Proof of it is given below.

Proof that the diagonal of the square in 45° oblique perspective is horizontal
Given the triangle LmR, in which RL is bisected at D by aD; mL is intersected at a by Ra; Kn is intersected at b by Lb; Rs and Lb intersect each other at c on aD;
To prove that ab is parallel to LR:
1. Extend aD and mark off DZ equal to De.
2. Since RD equals LD and DZ equals De and we know that the diagonals of a parallelogram bisect each other, RLcZ is a parallelogram.
3. Since bc and EZ are parallel, they divide the angle RnZ in the same ratio and nb/bR equals ne/ez.
4. Similarly, since ac and LZ are parallel lines, they divide angle LnZ in the same ratio, and ne/eZ equals na/AL.
5. Therefore nb/bR equals na/AL, and since lines nF and nL are divided in the same ratio, the segments mark off parallel lines.
Construction of a cube in 45° perspective

Since the diagonal of a cube is the same as the diagonal of a square, which can be constructed as follows:

Construction of a cube in 45° perspective

1. To construct a cube in 45° perspective, observe the following steps:
2. Draw a horizontal line, h, and choose a point, vp-L.
3. Bisect the line segment vp-L.
4. Draw a vertical line, v, from the midpoint of vp-L.
5. Draw a diagonal, N, from vp-L to vp-R.
6. Erect a vertical line, y, at the midpoint of the diagonal N.
7. Construct a square by rotating the diagonal, N, and drawing a new diagonal, x.
8. Construct a cube by connecting the vertices of the square, x, y, and z, to form a cube.

This cube is constructed using the method of drawing a cube in 45° perspective.
Since the diagonal of the square in 45° oblique perspective is parallel to the horizon and the picture plane it has no convergence. Thus it provides a constant measure which can be used to erect a cube.

Construction of the diagonal plane of a cube in 45° oblique perspective
To construct the diagonal plane of a cube, given the horizontal diagonal xy:
1. Erect the verticals at x and y.
2. Draw a diagonal line at 45° from x.
3. Rotate point y from x until it intersects the diagonal, locating point z.
4. Draw a horizontal through z.
Since xz equals xy, aawz is a side of the cube whose diagonal equals xy, and aawb is the diagonal plane of that cube.

Construction of a cube in 45° oblique perspective
To create a horizontal square in 45° oblique perspective, we need only apply the conditions observed in the diagram on the preceding page.
1. Draw a horizon and place two vanishing points vp-L and vp-R on it.
2. Bisect the distance between vanishing points to locate diagonal vanishing point DVP.
3. Drop a perpendicular or near perpendicular from DVP. This line is a diagonal of the square.
4. Draw two lines from vp-L and vp-R to intersect at the desired angle on the diagonal. This gives the nearest angle N of the square.
5. Draw two more perspective lines to intersect on the diagonal at the desired distance above N, enclosing the figure xynz.
xyzN is a horizontal square because it fulfills the visual conditions given on the preceding page:
a) The diagonal goes to DVP, which is half way between vp-R and vp-L.
b) Front and rear corners lie on the diagonal.
c) The side to side diagonal is truly horizontal (proof on previous page).
d) The four sides converge to their respective vanishing points.
We can easily complete the cube by erecting a diagonal plane on this square:
5. Erect verticals at all four corners of the square.
6. Construct the diagonal plane of the cube by rotating point y 45° upward to point z and drawing a horizontal through point z to intersect the side verticals (proof above).
8. Construct the upper square of the cube by drawing perspective lines through intersections.
This cube is absolutely accurate and can be checked against any constructional system. It is constructed directly from the horizon, without station points, elevations, long parallels or angles, measuring points, etc.
Since the front to back diagonal need not be vertical so long as it goes to the DVP (previous page), we have established a simple method of drawing an accurate cube in a variety of positions.
However, distortion will occur if the diagonal is too far from the vertical. On the basis of traditional perspective theory, there is no explanation for this distortion.
Example of the use of 45° perspective
This drawing of a range is based on a 45° view of the cube. The basic form, achieved by multiplication, is twice as high and twice as wide as the original cube. The 45° view is especially appropriate for an object with two interesting sides because it shows both sides with equal emphasis. Since the width of the range is greater than its depth, the view is not monotonous, but if the range were symmetrical another view would probably be more interesting.
In the last of method for perspective, we should be any direction drawing any draw a horizon perspective (plete vista of a few random distorted. The types of error if we want to first error is know how to complicated described.

Control of the
If an observat ally at eye a straight line nearest angle finally reaches at right an is seeing it for a plan vice thus it is im perspective t. Any square that angle is 90°. A simple way to swing a c the vanishing that any arc circle by line a circle and its. Thus any ne will be distor the circle wi

Side to side c
We have not found in squ vista again circle look d all right, but distorted as the original the accuracy paring them by the top pl accurate for
In the last chapter we discovered a simple method for erecting a cube in 45° oblique perspective. According to traditional theory, we should be able to multiply this cube in any direction and use it as a basis for constructing any figure. To see if this is true, draw a horizontal square in 45° oblique perspective (A1) and project it into a complete vista of horizontal squares. Now examine a few random squares and notice how many look distorted. These distortions result from two types of error, which we must learn to avoid if we want to make correct drawings. The first error is fairly simple, and most draftsmen know how to control it. The second is more complicated. To my knowledge it has never been described.

Control of the nearest angle
If an observer looks at a book held horizontally at eye level, the nearest angle is 180°—a straight line. As he lowers the book the nearest angle becomes more acute until it finally reaches 90°. When this happens, the book is at right angles to his line of sight and he is seeing it in plan. It is obviously impossible for a plan view to occur in a horizontal vista; thus it is impossible for any square in true perspective to have a nearest angle of 90°. Any square in a perspective vista whose nearest angle is 90° or less is distorted. A simple way of imposing this limitation is to swing a circle from the horizon to intersect the vanishing points. We know from geometry that any angle formed on the perimeter of a circle by lines from the intersections of the circle and its diameter will be a right triangle. Thus any nearest angle that touches the circle will be distorted; any nearest angle within the circle will be greater than 90°.

Side to side error
We have now eliminated the kind of distortion found in squares C, but if we examine our vista again we find many squares inside the circle look distorted. Those marked A look all right, but those marked B look increasingly distorted as their horizontal distance from the original square increases. We can check the accuracy of some of these squares by comparing them with squares drawn individually by the top plan method, which we know is accurate for drawing individual squares.
Comparison of correct squares with grid squares

In order to draw the top-plan squares that correspond with our grid squares, we must have an accurate plan view of each square.

Ordinarily, when we use the top-plan method, we start with a plan view and find the vanishing points when the perspective view is completed. But in this case, since we already know the vanishing points, we must work backwards from them to find the correct plan view. This is done as follows: Draw a perpendicular through the nearest angle of grid squares A, B, and C and locate the station point at the intersection of this perpendicular with a circle drawn through the vanishing points. Make a right angle at the station point by drawing lines to the vanishing points. Draw the nearest angle of the plan view square at the horizon with its sides parallel to the right angle. We are not particularly concerned with the size of the plan view squares; a convenient width can be found by projecting the next nearest angle of each grid square through the horizon. Now complete the plan-view squares and project them through the horizon to produce correct perspective squares over grid squares A, B, and C.

When we check squares A, B, and C against the corresponding top-plan squares we find that B and C accumulate error as they move away from the original, A. The nearest angles in each pair of squares coincide, and the sides are parallel; the significant difference is in the diagonals. Notice how diagonals 1, 2, and 3 change angle as they approach the vanishing point—in other words, the squares rotate as they move between the vanishing points.

Curved diagonals

Using the top-plan method, let us construct a series of horizontal squares across the circle of correct drawing and line them up along their diagonals. We find that the diagonals generate a curve. If we draw hundreds of accurate horizontal squares and line up their diagonals, the diagonals will make a figure like that at the bottom of the opposite page. A perfectly accurate drawing of any horizontal square in two-point perspective can be made on this figure simply by connecting perspective lines from the vanishing points so that they intersect on the diagonals.

The implications of this figure are startling: In a perfectly accurate perspective drawing, the only straight lines will be those that cross the observer’s line of sight. This may seem remarkable at first, but we can see it is so by the evidence of our eyes. Consider the case of an observer looking at a low building at right angles to his line of sight. The building will appear largest where it crosses his line of sight. At the sides of the vista it is further from him and will therefore appear to diminish. The roof, if it is above his line of sight, will curve down toward the horizon; the bottom, if it is below his line of sight, will appear to curve upward. Only if one of these lines is directly at eye level will it appear to be straight.
Let us construct a square across the face of the object, and let it be the same size as the original square. The diagonal of this new square will be perpendicular to the face of the object. If we draw lines from the corners of the new square to the corners of the original square, we will have a series of right triangles, each with a hypotenuse equal to the diagonal of the new square.

Once we have constructed these lines, we can use them to find the true size of the object. We will do this by measuring the length of each line and then calculating the length of the hypotenuse of each right triangle. We can then use these lengths to find the true size of the object.

We will find the true size of the object by using the Pythagorean theorem. The Pythagorean theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. In this case, the hypotenuse is the diagonal of the new square, and the other two sides are the sides of the original square. Therefore, we can write the equation as:

$$a^2 + b^2 = c^2$$

where $a$ and $b$ are the sides of the original square, and $c$ is the diagonal of the new square.

Once we have calculated the length of the hypotenuse, we can use this to find the true size of the object. We can do this by dividing the length of the hypotenuse by the length of the diagonal of the new square. This will give us the true size of the object.
It will help us to deal with error if we understand that accuracy is a relative thing in perspective. A drawing is a flat plane, and the eye registers impressions on a tiny, curved surface. Furthermore, perspective drawing is based on the assumption that the observer views any scene with one unmoving eye. It is true that the eye has a remarkably wide angle of vision: with both eyes looking straight ahead we can see almost a full 180°. But it would be difficult to draw what we see in this way. We gain an accurate impression of such a vista by moving our eyes constantly to look at each part directly. In the language of perspective, we see one scene with many horizons and vanishing points. Perspective drawing will always remain a system for symbolizing what the eye sees. Our aim is to make drawings that look accurate by learning to control the inevitable error.

Tolerable error

It would certainly be almost impossible to draw rectangular objects with curved lines, and our habits of seeing would make such a drawing look absurd. The curvilinear grid on the preceding page is not useful in drawing, but it does show us how distortion multiplies as side-to-side vista increases. We can apply this knowledge in two ways. If we are drawing a broad vista, we know we will have to tolerate some error, but we can learn to keep it within reasonable limits. If we are drawing a single object by developing an easily constructed view of the cube — and that is the basis of the system outlined in this book — we must find how far the cube can be extended before the error becomes intolerable. We saw on the previous page how a cube rotates on its diagonal according to its position between the vanishing points. If we wish to draw a cube at 45° to the observer, we must draw it half way between the vanishing points. The same construction can be used to draw a cube slightly to one side of center, but if the cube is considerably off center, the error in the diagonal begins to show up.

By comparing correct squares with grid squares, as we did on the preceding page, it is possible to determine the exact percentage of error that occurs at any point on a vista created by multiplying one square. At the right are two scales, one showing how error accrues in a vista developed from the 45° oblique view of a square, the other showing the error in a vista developed from a square correctly drawn at the 30-60° rotation. In order to find out how much error the trained eye could tolerate, I presented a test made up of correct and incorrect cubes to a large group of professional designers (left). I discovered that visual accord is assured if the error in depth does not exceed 25 percent. The 45° scale at right shows that an error of 25 percent occurs about half way to the vanishing points. In other words, the method presented for drawing a 45° view of the cube can be used to draw a cube at any rotation from the center half way to the vanishing points. At this point the error becomes intolerable and we need a view that is acceptable at the sides of the vista. Such a view is the 30-60° rotation.
error if we is a relative thing is a flat plane, and the on a tiny, curved perspective drawing is hat the observer viewing eye. It is true ally wide angle of king straight full 180°. But it what we see in this impression of such a constantly to look at language of one with many ints. Perspective in a system for see. Our aim is to occur by learning rror.

It is impossible to draw curved lines, and our ke such a drawing or grid on the ful in drawing, but it on multiples as s. We can apply this we are drawing a ill have to tolerate 1n to keep it within drawing a single adly constructed view the basis of the k — we must find tended before the We saw on the rotates on its diagonal between the vanishing a cube at 45° to the half way between same construction can right to one side is considerably off agoal begins areas with grid squares, g page, it is possible centage of error in a vista created by the right are two rror accues in a vista ligue view of a the error in a vista correctly drawn at er to find out how e could tolerate, p of correct and group of professional red that visual rror in depth does not scale at right shows occurs about half way 1 other words, the xing a 45° view of the a cube at any half way to the int the error becomes view that is the vista. Such a view
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We know t equals half LR equals Y is the mi
In the 30-6 since ls eq
We also know two sides of of the hypo LR; and if that 1 plus and since R: practical p
if LR equals
In short, we bisecting L.)

Construction
1. Draw a line through VP-L points V-P-L.
2. Bisect th points to loci.
3. Bisect th to locate A.
4. Bisect th locate measure.
5. Draw a line through N to angle N of t above or below.
6. Draw a line through N.
7. Lay off t and rotate Z X and Y.
8. Draw per the nearest a.
9. Draw a li. repeat for M. intersection
10. Draw pe at 1 and 2 to
11. Erect ve.
This cube is checked by an 45° oblique vi. at any rotation with an error error exceeds when it cross vanishing point, there is vanishing point in a two-point
Special case of 30-60° measuring points

Ordinarily, drawing a 30-60° view of the cube by the measuring point system requires the following preliminary construction: Draw a horizon and place two vanishing points on it (L and R); using L-R as the hypotenuse, construct a 30-60° right triangle with the apex at S; draw a vertical from S intersecting the horizon at A; rotate S to the horizon from L and R to locate the measuring points X and Y. At the 30-60° rotation it happens that the measuring points X and Y and the vertical at A can be located without construction of a right angle, by simple division of the hypotenuse (L-R).

We know that the short side of a 30-60° triangle equals half the hypotenuse; thus in LSR L-R equals 2LS, and since LS equals LY, Y is the midpoint of LR.

In the 30-60° triangle LAS, LS equals 2LA, and since LS equals LY, A is the midpoint of LY.

We also know that the sum of the squares of the two sides of a right triangle equals the square of the hypotenuse; thus LS² plus RS² equals LR²; and if LR equals 2, then LS equals 1; so that 1 plus RS² equals 4 and RS equals 1.732, and since RS equals RX, RX equals 1.732. For practical purposes, 1.732 becomes 1.70. Thus, if LR equals 2, RX equals 1.7.

In short, we can find Y by bisecting LR, A by bisecting LY, and X by bisecting LA.

Construction of a 30-60° view of the cube

1. Draw a horizon and establish two vanishing points VP-L and VP-R.
2. Bisect the distance between the vanishing points to locate measuring point MP-Y.
3. Bisect the distance between MP-Y and VP-L to locate A.
4. Bisect the distance between A and VP-L to locate measuring point MP-X.
5. Draw a vertical at A and place nearest angle N of the cube at the desired distance above or below eye level.
6. Draw a horizontal measuring line (ML) through N.
7. Lay off the height of the cube from N to Z and rotate Z to the measuring line, locating X and Y.
8. Draw perspective lines to N to construct the nearest angle.
9. Draw a line from MP-Y to Y and note its intersection with the perspective line at 1. Repeat for MP-Z and Z, locating 2.
10. Draw perspective lines to the intersections at 1 and 2 to complete the horizontal square.
11. Erect verticals at all corners of the square.
12. Draw perspective lines to Z, complete the cube. This cube is absolutely accurate and may be checked by any other system. It supplements the 45° oblique view, allowing us to draw a cube at any rotation between the vanishing points with an error of less than 2.5 percent. The error exceeds 2.5 percent in the 30-60° cube when it crosses the center or approaches the vanishing points. When it reaches the vanishing points, there is 100 percent error, so a vanishing point can never properly be included in a two-point perspective.
30-60° Example of the use of 30-60° perspective
This drawing of a dispensing machine was developed from a 30-60° view of the cube. The basic figure is four times as high and two and a half times as wide as the original cube. In practice, the final drawing would be made on tracing paper over the original construction so that no guide lines would appear. The 30-60° view is particularly appropriate for an object with one important side because it emphasizes one side while still showing the depth of the object and the configuration of a second side. If the interior of the machine were more important than the second side we would presumably choose parallel perspective.
PARA Construction of the cube in parallel perspective

Parallel perspective
We now have specific views of a 30°-60° view. With these methods of oblique perspective an oblique perspective is obtained. The vanishing point in a street scene is determined by the vanishing point of the street. This permits us to see the cube from one point above or below. A simplest form is still poorly as a case of 45° perspective. The tile floor is a square in parallel perspective two conditions are met.

Construction of:
1. Draw a horizontal line VP-L and VP-R.
2. Bisect the points to locate DVP. This is the vanishing point.
3. Draw perspective to create a horizontal line. The vanishing point is at 45° approach 90°.
4. Draw the horizon line. Draw diagonal lines. These four diagonal lines create the perspective. Again, we see the 45° square and parallel perspective.

Construction of parallel perspective:
1. Draw a horizontal line VP-L and VP-R.
2. Draw parallel line VP-L to find the depth.
3. Draw a perpendicular line VP-R to find the depth.
4. Draw a perpendicular line VP-L to find the depth.
5. Draw a line parallel to the horizon line.
6. Complete the perspective. Note side-to-side distortion at the edge.
Parallel perspective

We now have simple methods for drawing two specific views of the cube — the 45° view and the 30-60° view. We know that one or the other of these methods can be used to draw any oblique perspective view of the cube. However, an oblique perspective view can never include a vanishing point. In some drawings, particularly street scenes and interiors, the inclusion of a vanishing point is desirable because it permits us to show three side planes of the cube from within. A view which includes one vanishing point (or which has one vanishing point above or below the picture rather than off to one side) is in one-point or parallel perspective. Although it is the oldest and simplest form of perspective, parallel perspective is still poorly understood. It can be regarded as a case of 45° perspective, for the two forms are interlocked. We can see this by re-examining the tile floor that was used to illustrate 45° perspective. If we study the diagonals of the tiles, we find that they form squares in parallel perspective. The diagonals fulfill two conditions: the vanishing point of the front-to-rear diagonals appears in the center of the drawing, and the side-to-side diagonals are truly parallel to the horizon. Since the 45° square is easily drawn directly from the vanishing points, we can use it as a quick guide for drawing a square in parallel perspective.

Construction of a square in parallel perspective

1. Draw a horizon and place two vanishing points VP-L and VP-R on it.
2. Bisect the distance between the vanishing points to locate the diagonal vanishing point DVP. This is also the parallel perspective vanishing point.
3. Draw perspective lines from VP-L and VP-R to create a horizontal square BNCA, making certain that the nearest angle N does not approach 90°.
4. Draw diagonals through A and N parallel to the horizon.
5. Draw diagonals through C and B to DVP. These four diagonals form a square in parallel perspective. If we examine our tile drawing again we see that it is not necessary to complete the 45° square in order to draw a square in parallel perspective. A line drawn from one vanishing point through the middle of the 45° square will be the diagonal of the square in parallel perspective and serve to establish its depth (right).

Construction of a cube in parallel perspective

1. Draw a horizon and place two vanishing points on it. Bisect the distance between them to find the diagonal vanishing point.
2. Draw the front edge X-Y of the cube parallel to the horizon and rotate it upward 90° to Z to find the height of the cube.
3. Draw a perspective line from VP-L to Y.
4. Draw a perspective line from DVP to X intersecting the line from VP-L at W.
5. Draw a horizontal through W. This is the rear edge of the cube.
6. Complete the cube.

Less side-to-side shift can be tolerated in parallel perspective than in 30-60° or 45° perspective. Care must be taken to prevent distortion at extremes of the drawing.
Example of the use of parallel perspective

This drawing is developed from the largest perspective: the parallel vi of details of the object, but in other angles are uneventful. always give a object, but in the size
This drawing of a radio-phonograph was developed from a cube in parallel perspective. The largest possible cube was used to avoid the distortion that crops up so easily in parallel perspective: the basic box is twice the width and less than half the depth of the original cube. The parallel view was chosen because certain details of the interior would have been lost from any other angle, and the sides of the product are uneventful. Parallel perspective does not always give a clear indication of the depth of an object, but in this case the depth is not hard to judge: the size of the turntable is one clue.
Before we begin a drawing we must settle three things: the view, the scale of the object, and the size of the drawing. View, or the angle of rotation of object to observer, has been our subject so far. We now have simple methods for drawing three perspective views of the cube. Each method can be used to produce a range of views, so that by choosing the proper method we can draw an accurate cube at any angle to the observer. The preliminary construction in each case is simply a horizon and two vanishing points.

The 45° view
Place the diagonal vanishing point DVP half way between the vanishing points. Construct horizontal square YNZX with lines drawn from the vanishing points to intersect on a line from DVP. Raise verticals at corners. Rotate the diagonal of the square upward 45° to z and draw a horizontal. This gives us the diagonal plane YZBA of the cube. Draw perspective lines to complete cube.

The 30°-60° view
Bisect the distance between the vanishing points to locate one measuring point MP-Y; bisect the distance from MP-Y to locate the perpendicular X; and bisect the distance from X to locate second measuring point MP-Z. Draw perspective lines to construct the nearest angle N on X. Lay off the height of the cube NA and rotate it to a measuring line drawn through N, locating y and z. Draw lines from y to MP-Y and z to MP-Z, locating the side angles Y and Z. Complete the cube.

The parallel view
Bisect the distance between the vanishing points to locate diagonal vanishing point DVP. Draw the front edge of the cube YZ parallel to the horizon and rotate it up 90° to z to find the height of the cube. Draw perspective lines from VP-R to Y and Z. Find the diagonal XZ of the cube by drawing a perspective line from DVP. Complete the cube.
Any object can be rotated 90° between one set of vanishing points by using the parallel, 30-60°, 45° 30-60°, and parallel cases in order. In choosing the one view that will show an object to best advantage we can be guided by several considerations. 45° perspective serves well if the object is wider than it is deep; it gives a good view of both sides without being monotonous. 30-60° perspective helps avoid monotony if depth and width are almost equal, and is useful where one side contains most of the interest. Parallel perspective is used mainly for interiors and street scenes, but may be used for objects if they are very long and must fit a narrow frame.
Scale
One advantage of perspective drawings is the impression they give of the scale of objects. The draftsman should know how to control his drawing so that this impression is an accurate one. Scale, in perspective, is a factor of eye level and convergence. It can be assumed that all objects fall into three categories—small (clocks, jewelry, etc.), medium (automobiles, furniture, etc.), and large (buildings, ships, bridges). Convergence will be slight in small objects because they intercept so little of our cone of vision. It will be greater in medium-sized objects and considerable in large objects. Since the eye level of the observer is normally about five feet, it will usually be well above small objects, near the top of medium-sized objects, and near the base of large objects.

Thus a small object will ordinarily have the horizon high on the board and the vanishing points far apart in relation to the drawing. For medium-scale objects, the vanishing points will be closer together, and since they usually straddle the horizon, it will be slightly above the center of the drawing. Large objects are most impressive with eye level near the base of the drawing and the vanishing points close together to give a good deal of convergence. Sometimes, of course, the scale is intentionally upset to achieve dramatic effects.

Size
The size of the drawing depends directly on the distance between the vanishing points. Since the distance between the vanishing points is also a function of scale, a large drawing of a small object means a very large distance between the vanishing points. With a little practice, the draftsman will be able to judge how far the vanishing points should be spread and how high the horizon should be to achieve the proper scale and size for any object.

If he is skilled at freehand drawing he may lay out the cube by eye and derive the vanishing points from this sketch before proceeding with the mechanical drawing. Drawings can be enlarged by spreading the vanishing points, or by photostating a finished perspective if this is inconvenient.

In setting up a drawing, it is best to decide the view first and set up the cube in proper scale, then check to see if it is the proper size. If the cube is deficient in view, scale, or size it should be corrected at the outset. It is easier to redraw a faulty cube than to salvage a finished drawing.

After the basic drawing is complete, the original cube can be used as a rule for determining size and scale. 1. Extend the top edge of the cube to the common side of the square and draw a line from the corner to the common side. 2. Draw a line from the corner to the common side. 3. From an open corner of the cube, draw a line for the perspective lines.
After the basic perspective cube has been drawn it can be used as a measuring device to obtain a figure of any depth, width, and height. This is accomplished by multiplying and dividing the original cube.

**Multiplication of the cube**
1. Extend the sides in the direction of the new cube or cubes.
2. Choose any square $ACDB$ of the original cube that is in line with the proposed cube and bisect the common side $DB$ by drawing the diagonals of the square to locate its perspective center $X$ and drawing a perspective line through $X$ to the common side, locating $M$.
3. From an opposite corner $C$ of the square draw a line through $M$ to the extended side. The intersection with the extended side at $Bz$ is the depth of the next cube.
4. Complete the new cube with the necessary perspective lines and verticals.

**Division of the cube**
1. Find the perspective midpoint of any square of the cube by drawing the diagonals.
2. Draw perspective lines through the perspective center to divide the square into four equal squares.
3. Repeat as often as necessary to complete the division.
If very small divisions are required, the following is a quicker method:
1. Mark off the divisions on a vertical of the cube and draw perspective lines through them.
2. Draw two diagonals of the cube to intersect the perspective lines and draw verticals through the intersections.

Multiplication and division may be repeated as often as desired to create any combination of dimensions. In every case, the artist must determine which method is best from the proportions of the object he is drawing. Multiplication would be best if the object were $10 \times 1 \times 1$, for example, while division would be quickest if it were $10 \times 10 \times 1$. If the object is almost a cube, it is easier to draw the largest mass first and divide to find whatever irregularities may occur. In general, division is safer and may prevent exceeding the limits of accurate drawing.
Examples showing the development of a drawing

Example 1: a large object
The design of a truck has been settled to some extent in a rough. We want to convert this into a tight and accurate perspective drawing. First, we must choose our view. We want to show something of the front of the truck but are most interested in the side, and a 30-60° view seems the logical choice. Next, we consider the scale of the truck. Our drawing is intended to describe it accurately rather than dramatize its size. Presuming that the truck is about 12 feet high, we decide to place its midpoint slightly above eye level. It will fill as much of the zone of vision as possible. Finally, we decide on the best size for the finished drawing. We do not want the drawing to be cumbersome, but it must be of workable size for the amount of detail that will be included. In this case, we plan a drawing of medium size, about 18 inches across. Having made these preliminary decisions, we start by drawing a horizon fairly low on the table and placing two vanishing points on it 24 inches apart. Now we draw our basic 30-60° cube. The basic cube should be the largest unit that can be multiplied conveniently. We must watch out particularly for three errors: the final form must not cross the midpoint between the vanishing points; it must not come too close to the left-hand vanishing point; and the nearest angles at top and bottom must not approach 90°. We know that the ratio of the truck’s width to its length is 3:5; therefore the largest convenient cube will be a third of the width and a fifth of the length. This cube is multiplied to form a grid the over-all size of the truck, and details are added by further multiplication and division. (Rounded forms will be discussed in the next chapter.)

Example 2: a pencil sharpened smaller than the detail, the draw we wish it to be In order to achieve scale, we must 1 eye level and 12 apart. To begin near the top of the vanishing point decide on a 45° both sides well, since the object is wide. A cube of the sharpene horizon, and the proportions is (This demonstra be carried much mechanisms in the or down to extrem
Example 2: A small object
We start with two orthographic views of a pencil sharpener. Although the subject is smaller than the previous one and requires less detail, the drawing is for presentation and we wish it to be approximately the same size. In order to achieve the proper sense of scale, we must place the object well below eye level and spread the vanishing points far apart. To begin with, we set up a horizon near the top of the board and place the vanishing points about 48 inches apart. We decide on a 45° view because it will show both sides well, yet will not be monotonous since the object is about twice as long as it is wide. A cube equal to the desired width of the sharpener is constructed from the horizon, and from it a figure of the proper proportions is developed.
This demonstration, like the one opposite, could be carried much further. In order to show the mechanisms inside the upper half, for instance, the original perspective grid could be broken down to extremely fine dimensions.

Neither of these drawings was based on a predetermined scale situation: that is, a given size of drawing, a given scale of observer, or a given distance from eye level. This is easily accomplished, however, and is explained at the back of the book.
Circular forms
Any object, rectangular or circular forms, can be broken down into rectangular forms. To draw rectangular forms in elevation, follow these steps:
1. Identify the center of intersection of the four sides of the figure.
2. The circle is inscribed in a square.
   - The square is inscribed in the circle.
   - The edges of the square will fulfill the perspective conditions to a certain extent.

Construction of a circle in perspective
1. Draw any two diagonals.
2. Draw the center to bisect the diagonals.
3. Draw a circle through the points of the sides. This circle will be a perspective circle.

The ellipse
Ordinarily, we represent a smooth curve, curve inscribed in a square. The ellipse is a simple way of inscribing a circle in perspective. To draw ellipses or ellipses of perspective centers of a line. This line, a line of ellipses, crosses ellipses is thus.

Now draw two circles and locate their centers. If the major axis of the perspective ellipse is parallel to the ellipse, the perspective ellipse is drawn on a perspective plane. To use the ellipse, choose the midpoint of all ellipses, and locate the ellipse on a square, and locate the ellipse on the ellipse.

The drawing on the right shows why the ellipse is accurate at the center of the square. In perspective, the distortion increases from side to side. The ellipse's major axis is perpendicular to the square, and the minor axis is parallel to the square.
Circular forms

Any object, regardless of its shape, can be broken down into combinations of circular and rectangular forms. We have already learned how to draw rectangular forms. To understand circular forms, examine a circle drawn in a square in elevation. We notice two things:

1. The center of the circle coincides with the intersection of the diagonals of the square (O).
2. The circle is tangent at the midpoint of the four sides of the square (w, x, y, and z).

The square is the only rectilinear figure which will fulfill these two conditions. To draw a perspective circle, we need only transpose these conditions to a perspective square.

Construction of a circle in perspective

1. Draw any perspective square ABCD.
2. Draw diagonals to locate the center.
3. Draw perspective lines through the perspective center to bisect the sides (w, x, y, and z).
4. Draw a smooth curve tangent to the midpoints of the sides. This curve will be a circle in perspective.

The ellipse

Ordinarily, we would use a French curve to draw a smooth curve. However, it can be shown that any curve inscribed in a perspective square as described above is an ellipse. If we can find a simple way of matching an ellipse to a perspective square, we can use an ellipse guide for drawing perspective circles. The ellipse has two dimensions that may be useful, a major axis and a minor axis.

Draw ellipses on opposite faces of a perspective cube and connect the perspective centers of these faces with a perspective line. This line, which is perpendicular to both ellipses, crosses them at their narrowest dimensions and is thus the minor axis of both.

Now draw two concentric circles in perspective and locate their major and minor axes. Notice that while the minor axes are on the same line, the major axes (m and M) are erratic.

According to those two figures, the major axis of the ellipse is of no value in perspective drawing. The minor axis, on the other hand, is easily located: we need only find the perspective center of the square and draw a perspective perpendicular at this point.

Parallel circles have minor axes on a line.

To use the ellipse guide in drawing a perspective circle, choose the ellipse that is tangent at the midpoint of all four sides of the perspective square, and whose minor axis coincides with the perspective perpendicular.

The drawing opposite illustrates two reasons why the ellipse guide may not fulfill both these conditions at the same time:

1. An accurate ellipse will not fit a distorted square. In previous chapters we have seen how distortion increases as the cube is shifted from side to side.
2. Ellipse guides usually come in increments of 5°; the required ellipse may fall somewhere in between.
We have seen that a perspective circle is an ellipse inscribed in a perspective square so that it is tangent at the midpoint of the sides of the square and its minor axis is the perspective perpendicular of the square. If we find that the ellipse guide does not provide an ellipse that fulfills these two conditions, we have two choices: We can use the ellipse that comes closest to fitting the square when its minor axis is properly lined up, or we can draw a smooth curve tangent to the midpoints of the sides of the square, overlooking the probable error in minor axis. Generally speaking, the tangent-drawn curve will look better than the true ellipse, but it may be unwieldy to produce, and the draftsman must decide in each case what is the best method for drawing an acceptable perspective circle.

Suppose, for example, that we are drawing an automobile wheel. If the front wheel is in true perspective, it is not unusual for the rear wheel to be slightly in error. This wheel may involve as many as fifteen concentric ellipses, and it obviously will save time if we can use the ellipse guides. After we have drawn the original perspective square (A), we can see how much error there is by drawing a tangent curve ellipse (B) and comparing its minor axis with the perspective perpendicular. In practice we would disregard the small error and use the ellipse guides. We choose the ellipse that fits best when the minor axis is properly aligned (C) and finally complete the wheel by lining up a series of concentric ellipses along the same minor axis (D).
The twelve-point ellipse
Occasionally it is necessary to draw an ellipse larger than those found on ellipse guides. As an aid to drawing a smooth curve in such cases the draftsman can locate eight additional points on the ellipse by geometric construction. To see how this is done, let us start with a circle in a square:
1. Draw a square $ABCD$ and draw the diagonals to locate its center. Draw a circle tangent to the sides of the square.
2. Divide the square into quadrants by connecting the midpoints of the sides.
3. Locate the center of each quadrant by drawing the diagonals.
4. Draw horizontals and verticals through the center of each quadrant ($mm$ and $nn$, for example).
5. Draw line $Cm$. Its intersection with $nn$ will lie on the circle. Similarly, the intersection of $Cn$ with $mm$ will lie on the circle. By repeating this construction, eight points on the circle can be located in addition to the original tangent points.

When this construction is carried out with a perspective square, it locates eight points on the ellipse inscribed in that square.
Rotation
The ellipse is used to rotate planes or solid figures which are turned in relation to the basic perspective, such as pullman chairs, steering wheels, house roofs, box lids, etc. When an object is rotated on the horizontal plane the vanishing point or points are shifted along the horizon. To illustrate horizontal rotation, we will rotate a plane:
1. Construct a perspective square around the center of rotation of the plane and inscribe an ellipse in it. Construct additional ellipses intersecting all important points on the plane.
2. On the initial ellipse, plot the amount of rotation desired by construction or by judgment. From this point on the ellipse draw a line through the center of rotation to the horizon, locating the new vanishing point VP-Z.
3. Complete the new figure using the new vanishing point.

Rotation in a vertical plane is more complex because it usually throws the object into three-point perspective. The new vanishing points are directly above or below one of the original vanishing points. To illustrate vertical rotation, we will rotate a cube 45°.
1. Draw a vertical through VP-L.
2. Inscribe an ellipse on side ad of the cube.
3. Where the ellipse crosses diagonal ab of the side, draw a tangent b'd' and extend it until it intersects the vertical, locating VP-X. VP-X is the vanishing point of b'd' and its parallels.
4. Where the ellipse crosses diagonal c'b' draw a tangent to locate VP-Y.
5. Complete the cube, using vanishing points VP-X, VP-Y, and VP-R.

Four forms
The four basic forms of which all objects are composed are the cube, the cylinder, the cone, and the sphere. By applying ellipses to the basic rectilinear form, we generate the remaining three forms.
The cylinder and the cone require that a circle be drawn on one or both ends of a rectilinear figure. The sphere is unique because in any perspective it remains a circle in outline.
The ellipses are its equators; its outline is drawn from the perspective center of the cube and is tangent to the extremities of the ellipses.
Objects are outlined in the cone, or the cone is outlined in the objects. A red line is a vertical line, and a blue line is a horizontal line.

1. Locate the point where the cone intersects the plane of the object.
2. Rotate the cone around the point of intersection to create a new plane.
3. Project the new plane onto the original plane to create a new object.

The new object will have the same shape as the original object, but its orientation will be different. The new plane will intersect the original plane at a new angle, creating a new object that is similar to the original object.
The best way to draw compound forms like those shown here is to develop a perspective grid on which the curves can be plotted from one or more scale drawings. The example at right is a fairly simple one because the cross section is circular and does not need to be derived from a scale drawing. We start with a longitudinal section showing the free curving side in true scale and translate this curve to a perspective grid. Details are developed by multiplication and division in subsequent tracings. Finally, the circular cross-section is derived by constructing a series of ellipses tangent to the free curve with their centers on the main axis of the figure. The final outline is at the extremities of these ellipses. An object composed of free curves in more than one plane, like that at left, requires a complex grid in three dimensions. In this case the final outline was traced on a grid made by superimposing four horizontal sections on a vertical section. Usually only a few curves need be plotted accurately and the rest can be filled in by eye.

When he is designing directly in perspective, the designer can plot some basic curves on a perspective grid and try out variations on an overlay sheet. The system is invaluable where curved forms must envelope a given mechanism — in an electric shaver, for instance. In such cases, grids are drawn at critical points to make sure the curved form clears the mechanism.
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Three-point perspective

In ordinary two-point perspective, only two vanishing points are required. Some of the lines in the cube converge toward one of these vanishing points, some toward the other. A third set of lines — usually the vertical ones — do not converge at all but are parallel to each other. Their vanishing point is assumed to be at an infinite distance from the observer. In three-point perspective, a finite vanishing point is provided for all three sets of lines in the cube.

Three-point perspective should be used a good deal more than it is. Parallel lines appear parallel in perspective only if they happen also to be parallel to the picture plane, and in most positions the cube has no edges parallel to the picture plane. In other words, two-point perspective can be regarded as a special case of three-point perspective. It is used more than three-point perspective mainly because the latter is so much more difficult.

To understand three-point perspective examine this case: We want to draw a group of cubes suspended at various points between eye level and the plane view. Since the observer is looking squarely at the top cube, its verticals are perpendicular to his line of sight and parallel to the picture plane, and they appear parallel in perspective. If these verticals are parallel all the others must be; thus all four cubes are in two-point perspective. If the four cubes are lined up with their nearest edges against the picture plane, all the nearest edges are the same height, regardless of their relation to the observer.

Now suppose we are drawing four separate pictures of these four cubes. In this case the observer doesn’t have to maintain a fixed position, and he naturally turns toward each cube as he draws it. Since by definition the picture plane is perpendicular to the observer’s line of sight, it rotates to a new position for each cube. The top cube is still in two-point perspective, but the second and third cubes are at random angles to the picture plane; they are in three-point perspective.

The second drawing shows that a cube whose sides are perfectly vertical is in two-point perspective when it is straddling the horizon. If it is below the horizon (as most small objects are) it is seen at an angle and there is convergence in every pair of lines. Returning to the first drawing, we can easily see how the two-point cube grows more and more distorted as it drops away from the observer’s line of sight.

The draftsman often permits considerable distortion rather than introduce the third vanishing point because three-point perspective is complicated and space-consuming. In addition to a large horizontal area it usually requires tremendous depth for a third vanishing point. The drawings at the left compare two-point and three-point drawings of one object seen from one angle. Although the second drawing is more dramatic, the convergence is slight in an object so close to eye level, and we do not object to the distortion in the first drawing. Vertical convergence is more important in objects that are taller than they are wide. It is imperative in objects so far below eye level that more of the top is seen than the sides.
The only two of these vanishing points are those of the lines of sight to the vanishing point. A third point is not needed in our case because the picture is looking at an infinity. A third set of parallel lines in the picture would be more than would happen in a real-world setting, and in most cases because the line of sight is not parallel to the picture at an infinity point.

Returning to the example of cubes, we see that a cube whose side is small enough to be on the picture's line of sight is more distorted than if it were on the picture's line of sight. In addition to the third vanishing point, there is another vanishing point at the top of the picture on a line parallel to the picture's line of sight. This vanishing point is called the vanishing point of the top of the picture. The vanishing point of a line parallel to the picture's line of sight is called the vanishing point of the top of the picture. The vanishing point of a line parallel to the picture's line of sight is called the vanishing point of the top of the picture.
A cube in three-point perspective is derived from three horizons. These horizons are the three sides of an acute triangle. The vertices of the triangle are vanishing points. Its altitudes are measuring lines perpendicular to each horizon, and their intersection marks the nearest angle of the cube.

Construction of the cube in three-point perspective
1. Draw a horizon and place two vanishing points VP-L and VP-R on it.
2. Bisect the distance between VP-L and VP-R and from midpoint as draw a circle through them.
3. At any point X between the vanishing points, depending on the view desired, draw a vertical measuring line; its intersection with the circle locates the station point SP-X.
4. Draw lines from VP-L and VP-R to intersect on the vertical measuring line, locating nearest angle w at the desired distance below eye level.
Continue these lines to the circle, finding points Y and Z.
5. From VP-L draw a line through Z to the vertical measuring line. Repeat for VP-R and Y. The two lines will meet on the vertical measuring line at the third vanishing point VP-3. We now have three horizons, each with its vanishing points and its measuring line (at X, Y, and Z). We found the station point for one horizon on a circle drawn through the vanishing points. To find the other two station points we draw similar circles.
6. Bisect the horizon between VP-R and VP-3 and draw a circle through them. Station point SP-Y will be found where this circle intersects the measuring line through Y.
7. Repeat for the horizon between VP-L and VP-3, locating station point SP-Z.
8. Connect all the vanishing points with their station points, enclosing a cube.
9. Although this cube is accurate, it cannot be used because the corners at SP-X, SP-Y, and SP-Z are 90° angles and therefore at the limits of true perspective. To make usable cubes or a grid, we must subdivide it.
This method is accurate for any view of the cube, but it is cumbersome. Usually the draftsman can save time by using one of several special cases and short cuts.

The perfect 45° view
In one case of three-point perspective the cube is perfectly symmetrical. This happens when it is rotated at 45° with respect to each of the three horizons. Then front corner and rear corner coincide on the observer's line of sight; lines converge toward each vanishing point at equal angles; and corresponding corners are at equal distances from the eye. Construction of this cube is quick and accurate.
1. Draw a circle and place the nearest angle w of the cube at its center.
2. From w draw three measuring lines at 120°, locating the three vanishing points, and coincidentally the three station points, on the circle. The front edges of the cube will lie on these lines.
3. Connect perspective lines at any point X on one of the measuring lines. Draw perspective lines to meet at the same angle on the other measuring lines, and the cube is completed.
is derived from the vertices. Its perpendicular to the vanishing points.

P-L and VP-R are vertical through them.

Draw a vertical with the P-X, P-R to intersect the line located nearest below eye level.

Finding the vertical Z to the VP-R and VP-3, the vanishing point VP-3.

With its vanishing line (ation point for the two stations.

P-L and VP-3 and VP-L and VP-3 intersect the line with their

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View of the view by the draftsman special

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3-PT  Special cases of three-point perspective

The combination 45° and 30-60° view
A second special case of three-point perspective is the combination of a 45° view toward one horizon and a 30-60° view toward another. This view is less monotonous than the straight 45° view, but somewhat more complicated to draw.
1. Draw a horizon and divide it according to the usual 30-60° method to find the vanishing points, the measuring points, and the vertical measuring line.
2. Rotate the distance between the vanishing points to the vertical measuring line to find VP-3.
3. Draw the bisector of this arc. Its intersection with the vertical measuring line places the nearest angle n.
4. Draw a horizontal measuring line through n and complete the 30-60° horizontal square as usual.
5. Complete another side of the cube by drawing lines from VPL and VP-3 to meet on the bisector. The bisector is the diagonal of this side, and since it is half way between two vanishing points, the side is in 45° perspective.
6. Complete the cube.

Three-point perspective by trained eye
The two short-cuts just given are a help in producing two specific views of the cube in three-point perspective. A third short-cut can be used to draw the cube in a variety of positions. Basically, it is a method for introducing vertical convergence into a cube drawn in two-point perspective. It is not a mathematically accurate method, but depends partly on skillful judgment.
1. Draw a cube A-B in two-point perspective at any angle close to eye level.
2. Draw diagonals to find its midpoint M.
3. Rotate one vertical edge inward until it looks right for the distance below eye level.
4. Where the new edge intersects its diagonal (C) draw a new bottom plane for the cube, using the original vanishing points. Draw all the sides in to it.
This system assures that the vanishing point is directly under the cube, and that the foreshortening of the vertical distance is at least roughly in proportion to the convergence. It is most useful for objects near eye level, where vertical convergence might not be worth the trouble of a more elaborate method. In objects well below eye level, where vertical convergence is essential, the skilled eye is not adequate, and the orthodox method must be followed.
Let perspective points and vanishing points be toward another. The vanishing point at line to find the vanishing polyhedron and the vertical line through the vanishing point set on the biector.
Visualization
No matter how thoroughly he knows mechanical perspective, the draftsman must always rely on his own eye. Usually his first step in making a drawing is to sketch the object at various angles to help in deciding the best view. Often the “best” view is a matter of opinion, and if someone else has to okay the drawing it is a good idea to know his choice before the final drawing is begun.

At left is a typical series of preliminary sketches for an electric shaver. Each has its advantages, and any one of them might be chosen. The first would be the simplest to make because it is actually a mechanical plan view, and although it does not show the cross section, it gives the most accurate view of the face. The second shows the three most interesting sides with equal emphasis. The third shows the same three sides but emphasizes the face. The fourth is essentially a plan view, like the first, with a bare indication of the cross-section added. The final view is the least informative but the most dramatic—a shaver’s eye view.

If he wants the most generally informative view the artist will probably choose the third. It is not an especially dramatic view to start with, but when the drawing is completed, the artist can turn it to find a more effective view. On the opposite page, it is shown upside down, so that the shaver seems to be floating in air instead of lying on a table. A round mat helps to spotlight it.

The cold eye
When he is constructing a perspective drawing the artist must check constantly to see that it looks right. The eye grows tired from looking at a drawing for a long time, but there are various ways of giving it a fresh outlook. One of the best is a diminishing glass, which makes the drawing much smaller, frames it, and gives the work an entirely new appearance, so that bad proportions or a poor choice of view show up immediately. A more complex way of checking the work, but a useful one if the drawing is to be reproduced in a smaller size, is to make photostats, particularly negatives. A simple way of refreshing the eye is to look at the drawing in a mirror, or, if the paper is sufficiently transparent, simply to turn it over. When the artist is drawing at a large board, and particularly when he is seated, he should put the drawing on the wall or stand on his stool from time to time so that he does not always see the drawing in perspective. This is particularly important when he is doing a large three-point perspective, since vertical convergence is especially likely to be distorted if the drawing is always viewed at an angle.
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Ordinarily the artist uses his trained eye to make certain that his drawing is accurate; occasionally, his judgment tells him to violate the normal rules of perspective. His ability to do this depends on his complete understanding of these rules and his artistic judgment.

Adding a vanishing point
The drawing at the right illustrates the introduction of a vanishing point in a two-point perspective view. According to the rules we have learned, this means that the drawing extends beyond the limits of accuracy; the vanishing point should only appear in parallel perspective. Yet the vanishing point is bound to appear in a vista as broad as this, and sometimes a certain section of a drawing—a street or a line of buildings—is more effective in parallel perspective. Inevitably, there is distortion in the area around the vanishing point. It is usually not noticeable in flat surfaces like a tile floor or a building facade, but it is painfully apparent in any solid object like the cube at far left in the drawing opposite. It is a good idea to plan the drawing so that there are no solid objects in this area. No matter how carefully he plans, the artist will probably find some eyesores that can only be overcome by trial and error—by his own judgment.

Exaggeration
The use of the vanishing point in two-point perspective is an extreme violation of normal perspective rules. A simpler example is the use of exaggeration for dramatic effect—making objects appear bigger or smaller than they are, or very close or very distant. This is done by manipulating convergence and the distance above or below eye level. If we want to make a truck look particularly imposing, for example, we might draw it very high against the horizon and place the vanishing points close together to exaggerate the convergence. The drawing is not strictly accurate, but it has the desired effect.
Practice in drawing freehand cubes

It is a mistake for anyone to feel he is skilled in perspective until he has mastered freehand drawing. Even mechanical perspective requires a trained eye to check inaccuracies and to judge view, size, and scale. Freehand drawing requires skill in judging all the factors that make up perspective drawing simultaneously. The most important and difficult mental control is the judgment of proportion. The mental processes that control accurate proportion on paper are the same as those that permit the designer to judge good proportion in objects. Thus freehand drawing practice tends to make the designer more sensitive and improve his taste in form, line, curves, etc.

The key to superior freehand drawing is to break through the impulse to copy—either actually or mentally—an outline shape, and to develop a truly three-dimensional concept. The cube is the basic form in freehand work, as in mechanical work. I believe it is best to begin freehand practice without any guides, such as vanishing points or horizons. As a first exercise I ask students to draw dozens of cubes in a variety of positions. These are then divided into four groups according to four basic errors. The student soon learns to recognize these errors immediately and avoid them while he is making the drawing.

1. Proportion. Carefully inspect each side of the cube for squareness. The most oblique sides are the most difficult to judge. It is a help to draw the diagonals of each side. If the nearest angle is at 45° to the observer, the diagonals should be horizontal and vertical; their proper relationship at other rotations can be learned by observing them in mechanical drawings.

2. Tilted horizon. Even though the verticals are really vertical on the paper, they are not necessarily vertical to the horizon. This error, which is eliminated in mechanical perspective by the use of instruments, can be a subtle and annoying source of trouble in freehand drawing. It is discovered by extending the sides of the cube to find the vanishing points and the horizon.

3. Nearest angle. In freehand drawing, as in mechanical perspective, the nearest angle must always be greater than 90°.

4. Convergence. Notice that the lines do not all converge at the same vanishing points, giving a warped look to one of the planes. Placing a straightedge on doubtful lines will quickly point out any error.
View scale and size in freehand drawing

Set up two vanishing points and draw many cubes between them. Notice how the relationship of the sides changes as the cubes rotate from one vanishing point to the other. This exercise is also helpful in teaching proportion: since all the other variables are controlled by the position of the vanishing points, the student can concentrate on achieving the proper proportions of the sides.

Add a third vanishing point and the parallel view to this exercise.
Scale and size
Draw cubes of approximately the same actual size to represent small, medium, and large objects. Notice how the impression of smallness or bigness depends on convergence and eye level. Details can be added to enhance the impression.

The size of the drawing is in direct proportion to the distance between the vanishing points. This exercise shows that if objects of varying size are to be represented in drawings of similar size the distance between the vanishing points must vary widely.
Complex forms in freehand drawing

Rectilinear forms
When the student can draw the cube with confidence in all positions, he is ready to try more complex rectilinear forms. These should be drawn directly by eye and then broken down by division to a basic cube that can be examined for accuracy. When the draftsman is able to perform this check continually and automatically as he works, he is ready to proceed to the true freehand drawing of any object.

I want to draw a jeep.
Draw a rectilinear form with the proportions of the jeep; then add details in perspective until the drawing is complete.
The ellipse
At the start, draw the ellipse inside a perspective square. After a good deal of practice, forms with circular sections can be drawn directly. The beginner will find that it helps to turn the paper so that the minor axis is vertical and draw the ellipse horizontally against it.

I want to draw a watch
Draw the ellipse directly if advanced enough. Or else draw a rectilinear form of the proper proportions and divide it up with the diagonals and the bisectors of the sides. It takes a real understanding of convergence in the circle to place the numerals properly.

An advanced exercise
As an exercise to climax these skills, practice drawing complex figures upside down. This exercise puts an immense strain on the draftsman's knowledge of freehand drawing.
Exercise in freehand drawing

The constant practice that is required to develop and maintain freehand skill can be turned into an amusing pastime. For example, choose an object — as small as a fountain pen or as big as a railway train — and make many drawings of it to see how you can change its appearance by varying view and scale.
True scale
True scale can be defined as a controlled relationship between three measurements: the distance from the observer to the object, the size of the object, and its distance below or above eye level. Existing systems for achieving true scale are complicated, and usually used only when necessary. But with my perspective system, true scale can easily become an everyday tool in perspective drawing.
Suppose, for example, that we wish to show a console radio in a display room. We can imagine that the machine would be about ten feet away; that it would be about 3' high and 2' below eye level. Obviously if we can base our preliminary construction on these measurements we will eliminate trial and error.
To set up the principles of true scale, let us reproduce this situation by the top plan system. First, draw a top plan of a 45° cube at a scale of 3 units to a side. Draw the horizon through the nearest angle, and draw a vertical measuring line perpendicular to the horizon. Measure 10 units down the measuring line to find the station point; measure 2 units down to find the top of the cube and 3 more to find its height. Draw lines from the station point parallel to the sides of the top plan cube to locate the vanishing points, and complete the cube.
Notice that in 45° perspective, the station point and the vanishing points are at equal distances from the top plan cube.
This means that we do not need to find the station point but can lay off this distance directly on the horizon.

True scale in 45° construction
1. Draw a horizon.
2. Lay off two vanishing points and bisect the distance between them to locate the diagonal vanishing point.
3. Divide the distance from the diagonal vanishing point to one vanishing point into units to equal the distance of the station point.
4. Draw the vertical measuring line and either project the units at 45° from the horizon or step them off on the measuring line to find the top and bottom of the cube.
5. Construct the diagonal plane of the cube to one side of VML.
6. Draw a diagonal from the lower far corner D2 to the opposite vanishing point, locating the depth of the cube X. Complete the cube.

True scale in 30-60° construction
In 30-60° construction the station point is similarly related to the vanishing points.
1. Establish the unit measure on the horizon from the VML to the nearest VP and project it down 60° to the vertical measuring line to find the height of the cube.
2. Divide the horizon in the usual way to find measuring points and complete the cube.

True scale in parallel construction
In parallel construction, as in 45° construction, the unit is the same on the horizon as it is on the vertical measuring line.
When the face has been completed, the depth is found by drawing the diagonal to DVP.
Scales for direct drawing
The artist who has to make a number of drawings using the same view and the same distance between the vanishing points can save a little time on original construction by using a perspective scale. Essentially, such scales use divisions of the horizon as a measure of foreshortening. They are easy to make in any size and view, and a set of them can be a handy tool. I do not recommend their use by anyone who does not understand perspective drawing.

To make one of these scales, construct a horizon and mark off two vanishing points an even number of units apart. Construct a four-unit cube with its nearest angle close to ninety degrees and multiply it to make a vista of one-unit squares. Place a strip of paper along the horizon and project the intersections of the squares up to it, numbering them to the right and left from zero at the vertical measuring line. Note the unit distance to the vanishing points at the top of the scale. To use the scale, draw a horizon and place the scale on it. Locate the vertical measuring line and the vanishing points as indicated on the scale. Mark off the true height of the object on the vertical measuring line; then project its depth down from the scale.

The lineaid
With ordinary difficulties to use it, it is very valuable in perspective drawing. The lineaid is a moveable arm position. If set in the book, it will position the vanishing points in the correct position. If set in the book, it will position the vanishing points in the correct position.

Suppose we have vanishing points on the arm and we want to find the true height of a cube. Make a scale point and an arm length on the lineaid. Then, draw a string from the lineaid to the arm length. The lineaid gives the desired points (this place two points). Make a scale point and a string for the lineaid. Now, a string from the arm length provides the true height of the cube.

Other vanishing points
In addition to the lineaid, there are other devices that can be made devices point off the lineaid. Cut a arm the inside of the arm outside. Care that its can be made distances not too large. Clamp a second vanishing point string between one end by moving the other end.

If a stiff enough string is used, the arm length can be used to scale the size of the object. The lineaid may be read and measured from a circle to get distances.

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The lineaid
With ordinary perspective systems the lineaid is
difficult to use, but with the system presented here
it is very valuable, making it easy to construct
a perspective view with one of the vanishing points
off the table. In other words, it allows us to use
an ordinary drawing board for perspective views
that would ordinarily require a tremendous
drawing area.

The lineaid is a long straight-edge ending in two
moveable arms that can be fastened in any
position. If the arms are placed against two pins
set in the board, the lineaid will move up and down
the board in a regular arc. The vanishing point
is at the center of this arc; its position depends
on the position of the arms. When the arms
are at right angles to the blade, the vanishing
point is at an infinite distance, and the lineaid is,
in effect, a T-square. As the arms are turned,
the vanishing point comes closer.

Suppose we wish to draw a 30°-60° cube with one
vanishing point off the table. First draw a horizon.
Set the arms of the lineaid at the proper angle to
give the desired distance between the vanishing
points (this is not hard to judge by eye) and
place two pins in the board (z and y) to hold
them. Make sure that the lineaid lines up with
the horizon, then rotate it down to the bottom of
the drawing. Drive one pin in the left vanishing
point and another against the lineaid at z. Stretch
a string from these two pins and line it up with
the lineaid and the horizon; the second vanishing
point (VP-R) will be found at the apex of the
string. Now divide the distance between the
vanishing points with a steel tape or simply by
folding the string to find the measuring points
and the vertical measuring line. Since the lineaid
supplies the right-hand perspective lines, we
have no further need for VP-R.

The lineaid is particularly useful for three-point
perspective, making it possible to draw a 45° cube
with all three vanishing points off the table. As
the illustration shows, three pairs of pins are set
in a circle to provide three positions for the lineaid.

Other vanishing point tricks
In addition to the lineaid there are various home-
devices for drawing with one vanishing
point off the board:
1. Cut an arc out of plastic or cardboard; fasten
the inside piece to back of the T-square. Fasten
the outside piece to the drawing board, taking
care that its center falls on the horizon. These
can be made in different sizes with their center
distances noted.
2. Clamp a piece of wood to the table to hold the
second vanishing point. Drive a pin into each
vanishing point and stretch a wire or tough
string between them, fastening the wire at
one end by means of a light spring or rubber band.
If a stiff enough wire is used, sketching can
be done directly against it.

Special machines for drawing in perspective are
interesting theoretically but usually very expen-
sive and not much more useful than a straight-
edge. I believe that the perspective system
I have described makes complex equipment and
special charts unnecessary.